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ORIGINAL PAPER

New Spectral Second Kind Chebyshev Wavelets Scheme for Solving Systems of Integro-Differential Equations

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Abstract In this paper, a spectral scheme based on shifted second kind Chebyshev wavelets collocation method (S2CWCM) is introduced and used for solving systems of integro-differential equations. The main idea for obtaining spectral numerical solutions of these equations is essentially developed by reducing the linear or nonlinear equations with their initial conditions to a system of linear or nonlinear algebraic equations in the unknown expansion coefficients. Convergence analysis and some illustrative examples included, to demonstrate the validity and the applicability of the method. Numerical results obtained are compared favorably with the analytical known solutions.

Keywords Second kind Chebyshev polynomials · Wavelets · Collocation methods · Systems of integro-differential equations

Mathematics Subject Classification 65M70 · 65N35 · 35C10 · 42C10

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Introduction

Spectral methods are one of the principal methods of discretization for the numerical solution of differential equations. The main advantage of these methods lies in their accuracy for a given number of unknowns (see, for example [1,19,20]). For smooth problems in simple geometries, they offer exponential rates of convergence/spectral accuracy. The three most widely used spectral versions are the Galerkin, collocation, and Tau methods. Collocation methods have become increasingly popular for solving differential equations, also they are very useful in providing highly accurate solutions to nonlinear differential equations (see, for example [8,9]).

The subject of wavelets has recently drawn a great deal of attention from mathematical scientists in various disciplines. It is creating a common link between mathematicians, physicists, and electrical engineers. Wavelets theory is a relatively new and an emerging area in mathematical research. It has been applied to a wide range of engineering disciplines; particularly, wavelets are very successfully used in signal analysis for wave form representation and segmentations, time frequency analysis and fast algorithms for easy implementation. Wavelets permit the accurate representation of a variety of functions and operators. Moreover, wavelets establish a connection with fast numerical algorithms (see [6,15]).

The subject of nonlinear differential equations is a well established part of mathematics and its systematic development goes back to the early days of the development of calculus. Many recent advances in mathematics, paralleled by a renewed and flourishing interaction between mathematics, the sciences, and engineering, have again shown that many phenomena in applied sciences, modelled by differential equations will yield some mathematical explanation of these phenomena [7,16,21].

Chebyshev wavelets have been developed in [13] to solve the fractional order differential equations. In [17,18], a cosine and sine (CAS) wavelets operational matrix of fractional order integration have been derived and used to solve integro-differential equations of fractional order.

One approach for solving differential equations is based on converting the differential equations into integral equations through integration, approximating various signals involved in the equation by truncated orthogonal series and using the operational matrix of integration, to eliminate the integral operations.

Special attentions have been given to applications of block pulse functions [5], Legendre polynomials [4], Chebyshev polynomials [11], Haar wavelets [10], Legendre wavelets [12].

The main aim of this paper is to develop a new spectral algorithm for solving systems of integro-differential equations based on shifted second kind Chebyshev wavelets. The method reduces the systems of integro-differential equations with initial conditions to a system of algebraic equations in the unknown expansion coefficients. Large systems of algebraic equations may lead to greater computational complexity and large storage requirements. However the second kind Chebyshev wavelets is structurally sparse, this reduces drasticaly the computational complexity of solving the resulting algebraic system.

The structure of the paper is as follows. In "Some Properties of Second Kind Chebyshev Polynomials and their Shifted Forms" section, we give some relevant properties of second kind Chebyshev polynomials and their shifted forms. In "Shifted Second Kind Chebyshev Wavelets" section, we develop a new shifted second kind Chebyshev wavelets collocation methods, also we ascertain the convergence analysis of the proposed scheme. As an application of S2CWCM, numerical solutions of second-order linear and nonlinear two-point boundary value problems are implemented and presented in "Solution of Systems of

Integro-Differential Equations" section. In "Numerical Results and Discussions" section, some numerical examples are presented to show the efficiency and the applicability of the presented algorithm. Some concluding remarks are given in "Concluding Remarks" section.

Some Properties of Second Kind Chebyshev Polynomials and their Shifted Forms

In the present section, we discuss some relevant properties of the second kind Chebyshev polynomials and their shifted forms.

Second Kind Chebyshev Polynomials

It is well known that the second kind Chebyshev polynomials are defined on $[-1, 1]$ by

$$U_n(x) = \frac{\sin(n+1)\,\theta}{\sin\theta}, \quad x = \cos\theta, \quad \theta \;\; in \;\; [0, \pi],$$

These polynomials are orthogonal on $[-1, 1]$, i.e.,

$$\int_{-1}^{1} \sqrt{1-x^2}\, U_m(x)\, U_n(x)\, dx = \begin{cases} 0, & m \neq n, \\ \frac{\pi}{2}, & m = n. \end{cases} \tag{1}$$

The following properties of second kind Chebyshev polynomials (see, for instance, [14]) are of fundamental importance in the sequel. They are eigenfunctions of the following singular Sturm–Liouville equation

$$(1-x^2)\, D^2\, \phi_k(x) - 3x\, D\, \phi_k(x) + k(k+2)\, \phi_k(x) = 0,$$

where $D \equiv \frac{d}{dx}$ and may be generated by using the recurrence relation

$$U_{k+1}(x) = 2x\, U_k(x) - U_{k-1}(x), \quad k = 1, 2, 3, \ldots,$$

starting from $U_0(x) = 1$ and $U_1(x) = 2x$, or from Rodrigues formula

$$U_n(x) = \frac{(-2)^n\, (n+1)!}{(2n+1)!\,\sqrt{1-x^2}}\, D^n \left[(1-x^2)^{n+\frac{1}{2}}\right].$$

The following theorem is needed hereafter.

Theorem 1 [14] *The first derivative of second kind Chebyshev polynomials is given by*

$$D\, U_n(x) = 2 \sum_{\substack{k=0 \\ (k+n)\ odd}}^{n-1} (k+1)\, U_k(x). \tag{2}$$

Shifted Second Kind Chebyshev Polynomials

The shifted second kind Chebyshev polynomials are defined on $[0, 1]$ by $U_n^*(x) = U_n(2x-1)$. All results of second kind Chebyshev polynomials can be easily transformed to give the corresponding results for their shifted forms. The orthogonality relation with respect to the weight function $\sqrt{x-x^2}$ is given by

$$\int_0^1 \sqrt{x-x^2}\, U_n^*(x)\, U_m^*(x)\, dx = \begin{cases} 0, & m \neq n, \\ \frac{\pi}{8}, & m = n. \end{cases}$$

The first derivative of $U_n^*(x)$ is given in the following corollary.

Corollary 1 [14] *The first derivative of the shifted second kind Chebyshev polynomial is given by*

$$D\, U_n^*(x) = 4 \sum_{\substack{k=0 \\ (k+n)\ odd}}^{n-1} (k+1)\, U_k^*(x). \tag{3}$$

Shifted Second Kind Chebyshev Wavelets

Wavelets constitute of a family of functions constructed from dilation and translation of single function called the mother wavelet. When the dilation parameter a and the translation parameter b vary continuously, then we have the following family of continuous wavelets:

$$\psi_{a,b}(t) = |a|^{-1/2} \psi\left(\frac{t-b}{a}\right) \quad a, b, \in \mathbb{R}, \quad a \neq 0. \tag{4}$$

Second kind Chebyshev wavelets $\psi_{nm}(t) = \psi(k, n, m, t)$ have four arguments where k, n can assume any positive integer, m is the order of second kind Chebyshev polynomials, and t is the normalized time. They are defined on the interval [0, 1] by:

$$\psi_{nm}(t) = \begin{cases} \frac{2^{\frac{k+3}{2}}}{\sqrt{\pi}}\, U_m^*\left(2^k t - n\right), \; t \in \left[\frac{n}{2^k}, \frac{n+1}{2^k}\right], \\ 0, \qquad\qquad \text{otherwise}, \end{cases}, \quad m = 0, 1, \ldots, M, \quad n = 0, 1, \ldots, 2^k - 1. \tag{5}$$

Function Approximation

A function $f(t)$ defined over [0, 1] may be expanded in terms of second kind Chebyshev wavelets as

$$f(t) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} c_{nm}\, \psi_{nm}(t),$$

where

$$c_{nm} = \langle f(t), \psi_{nm}(t) \rangle_{\omega} = \frac{8}{\pi} \int_0^1 \omega(t)\, f(t)\, \psi_{nm}(t)\, dt, \tag{6}$$

and $w(t) = \sqrt{t-t^2}$. If the infinite series is truncated, then $f(t)$ can be approximated as

$$f(t) \simeq \sum_{n=0}^{2^k-1} \sum_{m=0}^{M} c_{nm}\, \psi_{nm}(t). \tag{7}$$

Convergence Analysis

We state a theorem ascertain that the second kind Chebyshev wavelet expansion of a function $f(x)$ in $L^2_w[0,1]$, the space of square Lesbegue integrable functions, with bounded second derivative, converges uniformly to $f(x)$.

Theorem 2 [2] *A function $f(x) \in L^2_\omega[0,1]$, with $|f''(x)| \leqslant L$, can be expanded as an infinite sum of Chebyshev wavelets, and the series converges uniformly to $f(x)$. Explicitly, the expansion coefficients in (6) satisfies the following inequality*

$$|c_{nm}| < \frac{8\sqrt{2\pi}\,L}{(n+1)^{\frac{5}{2}}(m+1)^2}, \quad \forall m > 1, n \geqslant 0. \tag{8}$$

Solution of Systems of Integro-Differential Equations

In this section, we present a Chebyshev wavelets collocation method, namely, second-kind Chebyshev wavelets collocation method (abbreviation) to numerically solve the following Systems:

$$\begin{aligned} u_i'''(x) &= f_i(x) + F_i(x, u_1, u_1', u_1'', u_2, u_2', u_2'', u_3, u_3', u_3'') \\ &\quad + \int_0^a k_i(x,t) G_i(u_1(t), u_1'(t), u_1''(t), u_2(t), u_2'(t), u_2''(t), u_3(t), u_3'(t), u_3''(t))\, dt, \\ &\quad i = 1, 2, 3, \end{aligned} \tag{9}$$

subject to the initial conditions

$$u_i^{(r)} = a_{ir}, \quad r = 0, 1, 2, \quad i = 1, 2, 3. \tag{10}$$

Based on Eq. (7) we approximate u_i, $i = 1, 2, 3$ in terms of S2CWCM as follows:

$$u_{i,k,M}(x) = \sum_{n=0}^{2^k-1} \sum_{m=0}^{M} c_{i,n,m}\, \psi_{nm}(x), \quad i = 1, 2, 3, \tag{11}$$

where a_{ir} are known constants, $x \in [0,1]$, $k_i(x,t) \in L^2([0,1] \times [0,1])$, $i = 1, 2, 3$ are the kernels, $f_i(x)$, $i = 1, 2, 3$ are known functions, F_i, G_i, $i = 1, 2, 3$ are linear or nonlinear functions, and u, v, w are unknown functions.it is known that at $a = 1$ the integro equation called Fredholm Integro-Differential Equations while at $a = x$ it called Volterra Integro-Differential Equations.
Now we can define the residuals of the system (9) as follows:

$$\begin{aligned} R_i(x) &= \sum_{n=0}^{2^k-1} \sum_{m=3}^{M} c_{i,n,m} \psi_{nm}'''(x) - f_i(x) \\ &\quad + F_i \Bigg(x, \sum_{n=0}^{2^k-1} \sum_{m=0}^{M} c_{1,n,m} \psi_{nm}, \sum_{n=0}^{2^k-1} \sum_{m=1}^{M} c_{1,n,m} \psi_{nm}', \sum_{n=0}^{2^k-1} \sum_{m=2}^{M} c_{1,n,m} \psi_{nm}'', \\ &\quad \sum_{n=0}^{2^k-1} \sum_{m=0}^{M} c_{2,n,m} \psi_{nm}, \sum_{n=0}^{2^k-1} \sum_{m=1}^{M} c_{2,n,m} \psi_{nm}', \sum_{n=0}^{2^k-1} \sum_{m=2}^{M} c_{2,n,m} \psi_{nm}'', \end{aligned}$$

$$\left.\sum_{n=0}^{2^k-1}\sum_{m=0}^{M} c_{3,n,m}\psi_{nm}, \sum_{n=0}^{2^k-1}\sum_{m=1}^{M} c_{3,n,m}\psi'_{nm}, \sum_{n=0}^{2^k-1}\sum_{m=2}^{M} c_{3,n,m}\psi''_{nm}\right)$$

$$-\int_0^a k_i(x,t)G_i\left(\sum_{n=0}^{2^k-1}\sum_{m=0}^{M} c_{1,n,m}\psi_{nm}, \sum_{n=0}^{2^k-1}\sum_{m=1}^{M} c_{1,n,m}\psi'_{nm}, \sum_{n=0}^{2^k-1}\sum_{m=2}^{M} c_{1,n,m}\psi''_{nm}\right.,$$

$$\sum_{n=0}^{2^k-1}\sum_{m=0}^{M} c_{2,n,m}\psi_{nm}, \sum_{n=0}^{2^k-1}\sum_{m=1}^{M} c_{2,n,m}\psi'_{nm}, \sum_{n=0}^{2^k-1}\sum_{m=2}^{M} c_{2,n,m}\psi''_{nm},$$

$$\left.\sum_{n=0}^{2^k-1}\sum_{m=0}^{M} c_{3,n,m}\psi_{nm}, \sum_{n=0}^{2^k-1}\sum_{m=1}^{M} c_{3,n,m}\psi'_{nm}, \sum_{n=0}^{2^k-1}\sum_{m=2}^{M} c_{3,n,m}\psi''_{nm}\right) dt, \quad i=1,2,3. \tag{12}$$

Now we collocate (12) at x_s the first $2^k(M+1)-3$ roots of $\tilde{U}_{2^k(M+1)}(x)$ to get

$$R_i(x_s) = 0, \quad i = 1, 2, 3, \tag{13}$$

moreover the use of the initial conditions yields

$$\sum_{n=0}^{2^k-1}\sum_{m=r}^{M} c_{i,n,m}\psi_{nm}^{(r)}(0) = a_{ir}, \quad i = 1, 2, 3. \tag{14}$$

Now Eqs. (13)–(14) generate a system of equations in the unknown expansion coefficients $c_{i,n,m}$ which may be solved with the well-known Newton's iterative methods.

Numerical Results and Discussions

In this section, the presented scheme given in "Solution of Systems of Integro-Differential Equations" section is applied to solve linear and nonlinear system of integro-differential equations. The efficiency and the applicability of the proposed scheme are illustrated by solving some examples of Volterra and Fredholm integro-differential equations. All computations are performed using Mathematica 9.

Example 1 [3] consider the following non-linear system of Volterra integro-differential equations

$$\begin{aligned} u''(x) &= x + 2x^3 + 2(v'(x))^2 - \int_0^x ((v'(t))^2 + u(t)w''(t))\,dt \\ v''(x) &= -3x^2 - xu(x) + \int_0^x (xtv'(t)u''(t) + w'(t))\,dt \\ w''(x) &= 2 - \frac{4}{3}x^3 + (u''(x))^2 - 2u^2(x) + \int_0^x (x^2v(t) + (u'(t))^2 + t^3w''(t))\,dt \end{aligned} \tag{15}$$

subject to the initial conditions

$$\begin{aligned} u(0) &= 0, \quad u'(0) = 0 \\ v(0) &= 0, \quad v'(0) = 1 \\ w(0) &= 0, \quad w'(0) = 0 \end{aligned} \tag{16}$$

Table 1 Maximum absolute error for Example 1 using the proposed technique

k	M	E_u	E_v	E_w
	2	5.55×10^{-17}	2.78×10^{-17}	4.16×10^{-17}
0	3	5.60×10^{-17}	2.97×10^{-17}	4.51×10^{-16}
	4	1.22×10^{-16}	3.55×10^{-17}	6.68×10^{-16}

Table 2 The comparison of maximum absolute error between the numerical solution using our method and the solutions in [3]

Method in [3]	S2CWCM with $k = 0$ and $M = 4$
1.23×10^{-9}	6.68×10^{-16}

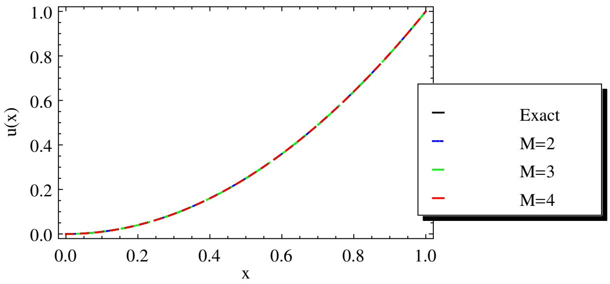

Fig. 1 The graph of the approximate solution $u(x)$ for Example 1 with different values of M.

with the exact solution $u(x) = x^2,\ v(x) = x,\ w(x) = 3x^2$. We solved the system (15) using the proposed technique S2CWCM. In Table 1, the maximum absolute errors, $E\_u,\ E_v$, and E_w, between exact and approximate solutions are listed for $k = 0$ and with various values of M, while in Table 2 we have compared the obtained results using the proposed technique with the obtained result in [3]. Moreover, Fig. 1 shows the approximate solution for u with different values of M.

Example 2 Consider the following non-linear system of Volterra integro-differential equations

$$\begin{aligned} u'''(x) &= x - u'(x) - \int_0^x ((u''(t))^2 + (v''(t))^2)\,dt \\ v'''(x) &= \sin x + \frac{1}{2}(\sin x)^2 + \int_0^x (u''(t)v(t))\,dt \end{aligned} \tag{17}$$

subject to the initial conditions

$$\begin{aligned} u(0) &= 0, \quad u'(0) = 1, \\ v(0) &= 1, \quad v'(0) = 0, \end{aligned} \tag{18}$$

with the exact solution $u(x) = \sin x,\ v(x) = \cos x$. Table 3 shows the maximum absolute errors, E_u and E_v, are listed for $k = 0, 1$ and with various values of M, while in Table 4 we have compared the obtained results using the proposed technique with the result obtained in [3]. Figures 2 and 3 show the approximate solutions for $u(x)$ and $v(x)$ respectively with various values of M.

Table 3 Maximum absolute error for Example 2 using the proposed technique

k	M	E_u	E_v	k	M	E_u	E_v
	8	3.64×10^{-8}	1.52×10^{-8}		7	3.64×10^{-9}	1.29×10^{-8}
0	9	7.28×10^{-10}	1.72×10^{-9}	1	8	5.59×10^{-10}	1.49×10^{-10}
	12	6.39×10^{-14}	2.92×10^{-14}		9	7.75×10^{-12}	2.31×10^{-11}

Table 4 The comparison of maximum absolute error between the numerical solution using our method and the solutions in [3]

Method in [3]	S2CWCM with $k = 0$ and $M = 8$
5.05×10^{-4}	3.64×10^{-8}

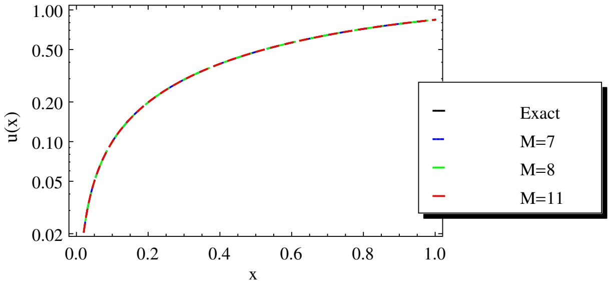

Fig. 2 The graph of the approximate solution $u(x)$ for Example 2 with different values of M.

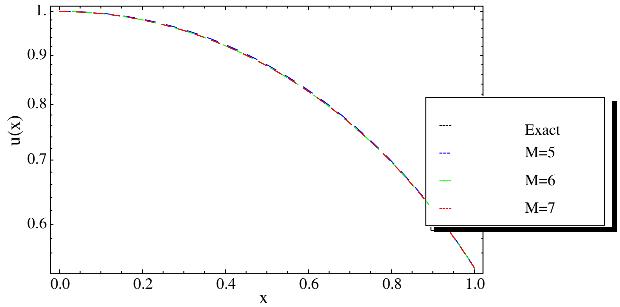

Fig. 3 The graph of the approximate solution $v(x)$ for Example 2 with different values of M

Example 3 [23] Consider the following linear system of Fredholm integro-differential equations

$$y'' + x\,y' - x\,y - \int_{-1}^{1} e^{-t}\ \sin x\ y(t)\,dt = e^{x} - 2\ \sin x \tag{19}$$

subject to the conditions

$$y(0) = y'(0) = 1, \quad -1 \leq x \leq 1 \tag{20}$$

with the exact solution $y(x) = e^{x}$. We convert Eq. (23) into a first order system of Fredholm integro-differential equations. Table 5, show the maximum absolute error, E_y, between the exact solution and the approximate solution obtained using the proposed technique S2CWCM for $k = 0$ and with various values of M, while in Table 6 the obtained result using the proposed

Table 5 Maximum absolute error for Example 3 using the proposed technique

k	M	E_y
	4	6.342×10^{-4}
0	9	2.553×10^{-9}
	13	1.455×10^{-13}

Table 6 The comparison of maximum absolute error between the numerical solution using our method and the solutions in [23]

Method in [23]	S2CWCM with $k = 0$ and $M = 10$
2.546×10^{-9}	4.600×10^{-10}

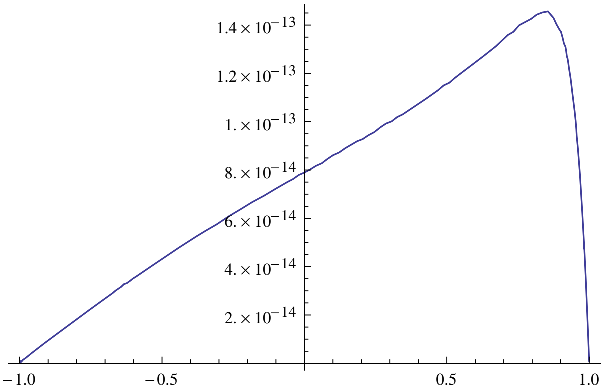

Fig. 4 Data evaluated for the absolute errors with $M = 13$ for Example 3

technique is compared with the result obtained in [23]. Moreover, Fig. 4 reveals the absolute errors with $M = 13$, using the proposed technique.

Example 4 [22] Consider the following non-linear system of Fredholm integro-differential equations

$$y^{(4)} - 2\,y^{(2)}\left(\pi^2 + \int_0^1 y'^2\,dt\right) + 4\,\pi^4\,\sin(\pi\,x) = 0 \tag{21}$$

subject to the conditions

$$y(0) = y''(0) = y(1) = y''(1) = 0, \tag{22}$$

with the exact solution $y(x) = -\sin(\pi\,x)$. We convert Eq. (21) into a first order system of Fredholm integro-differential equations. In Table 7, the maximum absolute error E_y is listed for $k = 0$ and with different values of M, while in Table 8 we have compared the results obtained by applying S2CWCM with the result obtained in [22]. Also, the absolute errors with $M = 16$ using the proposed technique are plotted in Fig. 5.

Table 7 Maximum absolute error for Example 4 using the proposed technique

k	M	E_y
	6	1.525×10^{-4}
0	10	4.125×10^{-8}
	16	1.315×10^{-13}

Table 8 The comparison of maximum absolute error between the numerical solution using our method and the solutions in [22]

Method in [22]	S2CWCM with $k = 0$ and $M = 12$
4.168×10^{-10}	9.552×10^{-11}

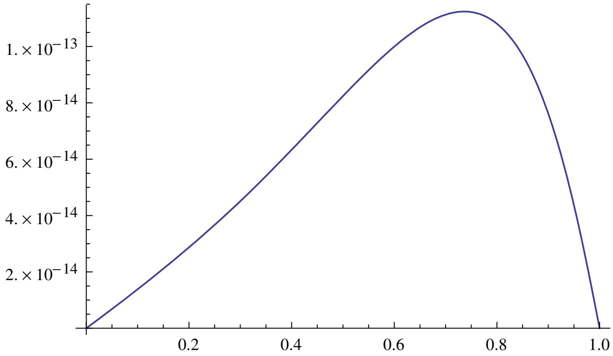

Fig. 5 Data evaluated for the absolute errors with $M = 16$ for Example 4

Example 5 Consider the following linear system of Fredholm integro-differential equations

$$\begin{aligned} u''(x) + v'(x) + \int_0^1 2xt(u(t) - 3v(t))\,dt &= 3x^2 + \frac{3}{10}x + 8 \\ v''(x) + u'(x) + \int_0^1 3(2x + t^2)(u(t) - 2v(t))\,dt &= 21x + \frac{4}{5} \end{aligned} \tag{23}$$

subject to the initial conditions

$$\begin{aligned} u(0) &= 1, \quad u'(0) = 0, \\ v(0) &= -1, \quad v'(0) = 2, \end{aligned} \tag{24}$$

the exact solutions are $u(x) = 3x^2 + 1$, $v(x) = x^3 + 2x - 1$. The system (23) are solved using S2CWCM in case of $k = 0$, and $M = 3$. After applying our technique, we get

$$c_{0,0} = 1.9375, \quad c_{0,1} = 0.75, \quad c_{0,2} = 0.1875, \quad c_{0,3} = 0, \tag{25}$$

$$d_{0,0} = 0.21875, \quad d_{0,1} = 0.71875, \quad d_{0,2} = 0.09375, \quad d_{0,3} = 0.015625, \tag{26}$$

by substituting these coefficients into Eq. (11) we get $u(x) = 3x^2 + 1$ and $v(x) = x^3 + 2x - 1$ which are the exact solutions (Fig. 6). Figure 6 shows the approximate solution for $u(x)$, with various values of M.

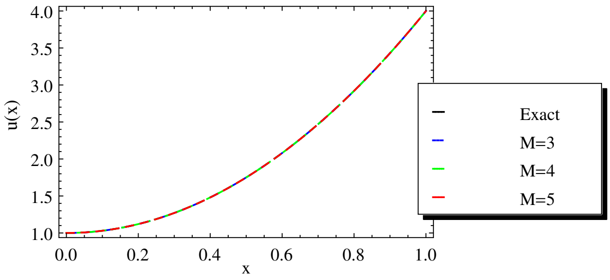

Fig. 6 The graph of the approximate solution $u(x)$ for Example 5 with different values of M

Table 9 Maximum absolute error for Example 6 using the proposed technique

k	M	E_u	E_v	k	M	E_u	E_v
	9	6.06×10^{-11}	2.02×10^{-10}		8	1.40×10^{-9}	4.80×10^{-9}
0	10	2.01×10^{-12}	7.54×10^{-12}	1	9	6.06×10^{-11}	2.02×10^{-10}
	11	7.04×10^{-14}	2.51×10^{-13}		10	5.83×10^{-11}	5.41×10^{-11}

Table 10 The comparison of maximum absolute error between the numerical solution using our method and the solutions in [3]

Method in [3]	S2CWCM with $k = 1$ and $M = 9$
5.05×10^{-4}	4.80×10^{-9}

Example 6 Consider the following non-linear system of Fredholm integro-differential equations

$$
\begin{aligned}
&u''(x) + 4v(x) + \int_0^1 2((x+t)u^2(t) + (2x-t)v'(t))\,dt = 4e^x + 2ex - \frac{2}{15}x + \frac{13}{6} \\
&v''(x) - 3u(x) + \frac{1}{2}\int_0^1 ((x-t)u'(t)V'(t) - (x+t)v^3(t))\,dt \\
&\qquad = e^x - 3x^2 + \frac{7}{6}x - \frac{1}{6}e^3x - \frac{1}{9}e^3 - e - \frac{19}{18}, 0 \leqslant x \leqslant 1
\end{aligned}
\tag{27}
$$

subject to the initial conditions

$$
\begin{aligned}
u(0) &= 1, \quad u'(0) = 0, \\
v(0) &= 1, \quad v'(0) = 1,
\end{aligned}
\tag{28}
$$

with the exact solution $u(x) = x^2 + 1,\ v(x) = e^x$. In Table 9, the maximum absolute error is listed for $k = 0, 1$ and with various values of M, while in Table 10 we have compared our results with the result obtained in [3]. Figure 7 shows the approximate solution for $u(x)$, with various values of M.

Remark 1 It is worthy noting here that the obtained numerical results in the previous solved examples are very accurate, although the number of retained modes in the spectral expansion is very few, and again the numerical results are comparing favorably with the known analytical solutions.

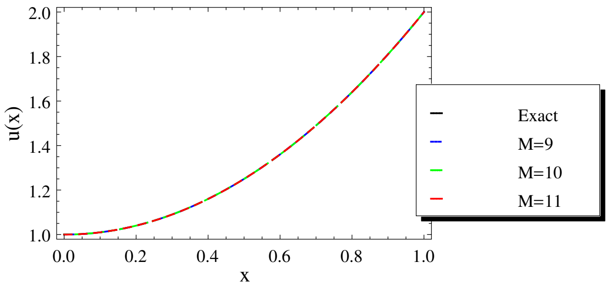

Fig. 7 The graph of the approximate solution $u(x)$ for Example 6 with different values of M

Concluding Remarks

In this paper, a new numerical scheme is presented to solve systems of integro-differential equations. The derivation of this scheme is essentially based on constructing the shifted second kind Chebyshev wavelets collocations methods. One of the main advantages of the presented scheme is its availability for application on both linear and non linear systems of integro-differential equations. Another advantage of the developed scheme is that, high accurate approximate solutions are achieved using a few number of the second kind Chebyshev wavelets. The obtained numerical results are comparing favorably with the analytical ones.

References

1. Abd-Elhameed, W.M., Doha, E.H., Youssri, Y.H.: Efficient spectral-Petrov–Galerkin methods for third- and fifth-order differential equations using general parameters generalized Jacobi polynomials. Quaest. Math. **36**, 15–38 (2013)
2. Abd-Elhameed, W. M., Doha, E. H., Youssri, Y. H.: New spectral second kind Chebyshev wavelets algorithm for solving linear and nonlinear second-order differential equations involving singular and Bratu type equations. Abstr. Appl. Anal. Vol. 2013. Hindawi Publishing Corporation (2013)
3. Biazar, J., Ebrahimi, H.: A strong method for solving systems of integro-differential equations. Appl. Math. **2**(9), 1105–1113 (2011)
4. Chang, R.Y., Wang, M.L.: Shifted Legendre direct method for variational problems. J. Optim. Theor. Appl. **39**, 299–307 (1983)
5. Chen, C.F., Tsay, Y.T., Wu, T.T.: Walsh operational matrices for fractional calculus and their application to distributed parameter systems. J. Frankl. Inst. **503**, 267–284 (1977)
6. Constantmldes, A.: Applied Numerical Methods with Personal Computers. McGraw-Hill, New York (1987)
7. N. Daoudi-Merzagui, F. Derrab and A. Boucherif, Subharmonic solutions of nonautonomous second order differential equations with singular nonlinearities, Abstr. Appl. Anal. **2012** , Article ID 903281, (2012). doi:10.1155/2012/903281
8. Doha, E.H., Abd-Elhameed, W.M., Youssri, Y.H.: Second kind Chebyshev operational matrix algorithm for solving differential equations of Lane-Emden type. New Astron **23–24**, 113–117 (2013)
9. Doha, E.H., Abd-Elhameed, W.M., Youssri, Y.H.: New wavelets collocation method for solving second-order multiPoint boundary value problems using Chebyshev polynomials of third and fourth kinds. Abs. Appl. Anal. **2013**, Article ID 542839, (2013)
10. Gu, J.S., Jiang, W.S.: The Haar wavelets operational matrix of integration. Int. J. Syst. Sci. **27**, 623–628 (1996)
11. Horng, I.R., Chou, J.H.: Shifted Chebyshev direct method for solving variational problems. Int. J. Syst. Sci. **16**, 855–861 (1985)
12. Khellat, F., Yousefi, S.A.: The linear Legendre mother wavelets operational matrix of integration and its application. J. Frankl. Inst. **143**, 181–190 (2006)

13. Li, Y.L.: Solving a nonlinear fractional differential equation using Chebyshev wavelets. Commun. Nonlinear Sci. Numer. Simul. **15**, 2284–2292 (2010)
14. Mason, J.C., Handscomb, D.C.: Chebyshev Polynomials. Chapman and Hall, CRC, New York, Boca Raton (2003)
15. Newland, D.E.: An Introduction to Random Vibrations. Spectral and Wavelet Analysis. Longman Scientific and Technical, New York (1993)
16. Pasic, M.: Fite-Wintner-Leighton-type oscillation criteria for second-order differential equations with nonlinear damping. Abstr. Appl. Anal. **2013**, Article ID 852180, (2013). doi:10.1155/2013/852180
17. Saeedi, H., Mohseni Moghadam, M., Mollahasani, N., Chuev, G.N.: A CAS wavelet method for solving nonlinear Fredholm integro-differential equations of fractional order. Commun. Nonlinear Sci. Numer. Simul. **16**, 1154–1163 (2011)
18. Saeedi, H.: Mohseni Moghadam, M.: Numerical solution of nonlinear Volterra integro-differential equations of arbitrary order by CAS wavelets. Appl. Math. Comput. **16**, 1216–1226 (2011)
19. Sweilam, N.H., Nagy, A.M., El-Sayed, A.A.: Second kind shifted Chebyshev polynomials for solving space fractional order diffusion equation. Chaos Solitons Fractals **73**, 141–147 (2015)
20. Sweilam, N.H., Nagy, A.M., El-Sayed, A.A.: On the numerical solution of space fractional order diffusion equation via shifted Chebyshev polynomials of the third kind. J. King Saud Univ. Sci. **28**(1), 41–47 (2016)
21. Turkyilmazoglu, M.: Effective computation of exact and analytic approximate solutions to singular nonlinear equations of Lane-Emden-Fowler type. Appl. Math. Model. **37**, 7539–7548 (2013)
22. Turkyilmazoglu, M.: High-order nonlinear Volterra-Fredholm-Hammerstein integro-differential equations and their effective computation. Appl. Math. Comput. **247**, 410–416 (2014)
23. Turkyilmazoglu, M.: An effective approach for numerical solutions of high-order Fredholm integro-differential equations. Appl. Math. Comput. **227**, 384–398 (2014)